

## PH120: Freshman Laboratory Guidelines on Graphing and Error Analysis

1. Whenever a quantity is measured, the absolute error should be estimated or determined from the equipment data sheet. For instance,  $D = 3.42 \text{ cm} \pm 0.02 \text{ cm}$  or  $f = 1025 \text{ Hz} \pm 5 \text{ Hz}$ . Usually, there is uncertainty in the estimation. Just use common sense and justify your decision.
2. The absolute error in a calculated result is computed as follows. Determine (using estimations, reason, or your data) the MAX value; this is the largest the value can be, taking into account the errors in the quantities involved in the calculation. Next compute the MIN value; this is the smallest the value can be based on the estimated or determined errors. The absolute error is given by

$$\text{Absolute Error} = (\text{Max Value} - \text{Min Value}) / 2$$

For example, suppose  $\text{speed} = (h_2 - h_1) / 100.0 \text{ s}$ . Suppose further that  $h_2 = 6.07 \text{ cm} \pm 0.02 \text{ cm}$  and  $h_1 = 1.67 \text{ cm} \pm 0.04 \text{ cm}$  and  $100.0 \text{ s}$  is accurate to  $\pm 0.1 \text{ s}$ . The MAX value is  $([6.07 \text{ cm} + 0.02 \text{ cm}] - [1.67 \text{ cm} - 0.04 \text{ cm}]) / [100.0 \text{ s} - 0.1 \text{ s}] = 0.0466 \text{ cm/s}$  using correct significant figures. The MIN value is  $([6.07 \text{ cm} - 0.02 \text{ cm}] - [1.67 \text{ cm} + 0.04 \text{ cm}]) / [100.0 \text{ s} + 0.1 \text{ s}] = 0.0434 \text{ cm/s}$

$$\text{The absolute error is } (0.0466 \text{ cm/s} - 0.0434 \text{ cm/s}) / 2 = 0.0006 \text{ cm/s}$$

3. The BEST value is given by

$$\text{Best Value} = (\text{Max Value} + \text{Min Value}) / 2$$

Using the example above the best value is  $(0.0466 \text{ cm/s} + 0.0434 \text{ cm/s}) / 2 = 0.0450 \text{ cm/s}$

4. Don't forget the rules of significant figures. When you add and subtract the result has the same number of digits as the number with the least number of digits (e.g.  $1021 \text{ m} - 0.0002 \text{ m} = 1021 \text{ m}$ ). When you multiply and divide the result has the same significant figures as the number with the worst significant figure (e.g.  $1021 \text{ m} * 0.0002 \text{ m} = 0.2 \text{ m}^2$ )
5. When calculating the MAX and MIN values don't forget the following rules

$$\text{MAX via addition} = (h_1 + \Delta h_1) + (h_2 + \Delta h_2)$$

$$\text{MIN via addition} = (h_1 - \Delta h_1) + (h_2 - \Delta h_2)$$

$$\text{MAX via subtraction} = (h_1 + \Delta h_1) - (h_2 - \Delta h_2)$$

$$\text{MIN via subtraction} = (h_1 - \Delta h_1) - (h_2 + \Delta h_2)$$

$$\begin{aligned} \text{MAX via multiplication} &= (h_1 + \Delta h_1) * (h_2 + \Delta h_2) \\ \text{MIN via multiplication} &= (h_1 - \Delta h_1) * (h_2 - \Delta h_2) \\ \text{MAX via division} &= (h_1 + \Delta h_1) / (h_2 - \Delta h_2) \\ \text{MIN via division} &= (h_1 - \Delta h_1) / (h_2 + \Delta h_2) \end{aligned}$$

6. You can represent error in three ways

- (a) Best Value +/- Absolute Error
- (b) Best Value +/- Fractional Error
- (c) Best Value +/- Percent Fractional Error

Where fractional error = absolute error / best value, and percent fractional error is 100% \* absolute error / best value.

7. By convention the absolute error is expressed to only one significant figure. When you calculate the error round it up to one significant figure (never, ever round down your error. Doing this reduces your error and this is why bridges fall down). Once you have determined the one significant figure for your error round your best value to the same digit. For example, it makes no sense to say  $X = 2.48 \text{ cm} \pm 0.5 \text{ cm}$ . Rather, one should report  $X = 2.5 \text{ cm} \pm 0.5 \text{ cm}$ . Another example, it is incorrect to report  $t = 105 \text{ s} \pm 0.4 \text{ s}$ . Rather, one should report  $t = 105.0 \text{ s} \pm 0.4 \text{ s}$ .
8. Frequently in science quantities are graphed with the expectation that a straight line (or a quadratic) will result. When such is the case, the slope and sometimes the intercept have meaningful values. Because the slope of a best fit line is in essence a graphical average its accuracy should be greater than the quantities plotted. We will deduce the error in the slope and intercept as follows.

The BEST slope is determined from the "best" straight line which can be drawn through the points. Selecting the best straight line fit is subjective; you must use your best judgement when doing a line fit by hand. Note: In some cases it is appropriate to use the origin as a data point and in other cases it is not appropriate.

The MAX slope is the steepest line which can reasonably be drawn through the points.

The MIN slope is the least steep slope which can be reasonably be drawn through the points.

The slope of the line with its error is given by

$$(\text{Max-slope} + \text{Min-slope}) / 2 \pm (\text{Max-slope} - \text{Min-slope}) / 2$$

When graphing by hand you want to determine the max best fit line and the min best fit line. The slopes and y-intercept of these two lines result in understanding the slope and y-intercept of the fits with error. Don't forget to use appropriate significant figures.

9. Quick reminder, the formula for a straight line is  $Y = m \cdot X + b$  ; where y is plotted on the y-axis, x is plotted on the x-axis, m is the slope, and b is the y-intercept.

## 10. Graphing Rules

- (a) Only use scientific graph paper (ask your instructor for "official" graph paper)
- (b) X and Y axis scaling must be picked so the smallest vertical and horizontal divisions are multiples of 1, 2, or 5 times a power of ten. *This is required by the scientific community, even if the graph paper is not efficiently used.*
- (c) Axes must be clearly labeled and you must include an instructive title (*position vs. time* is not an instructive title. *Determining the acceleration of a cart sliding along an inclined ramp* is an instructive title.)
- (d) When doing graphs by hand you must show your detailed slope calculations on each graph. When using software to create your graphs you must include the equation(s) for your line fit modified to include units and other necessary information.
- (e) Do not use your data points for slope calculations. Slope calculation points should be as far apart as possible. Select the appropriate axis scales (they need not be the same scale), plot your data, do your max and min line fits, pick the furthest data points on your lines that are still on your graph and use these points to calculate your slopes.
- (f) Include error bars on your graphs. Follow the examples given in class.